REPORT No. 395

A NEW PRINCIPLE OF SOUND FREQUENCY ANALYSIS

By Theodore Theodorsen

SUMMARY

In connection with a study of aircraft and propeller noises, the National Advisory Committee for Aeronautics has developed an instrument for sound-frequency analysis which differs fundamentally from previous types, and which, owing to its simplicity of principle, construction, and operation, has proved to be of value in this investigation. The method is based on the well-known fact that the ohmic loss in an electrical resistance is equal to the sum of the losses of the harmonic components of a complex wave, except for the case in which any two components approach or attain vectorial identity, in which case the ohmic loss is increased by a definite amount. This fact has been utilized for the purpose of frequency analysis by applying the unknown complex voltage and a known voltage of pure sine form to a common resistance. By varying the frequency of the latter throughout the range in question, the individual components of the former will manifest themselves, both with respect to intensity as well as frequency, by changes in the temperature of the resistance. This principle of frequency analysis has been presented mathematically and a number of distinct advantages relative to previous methods have been pointed out. Among these is the fact that the frequency discrimination or resolving power is inherently large and remains constant for the entire working range. No difficulties exist as to distortions of any kind. The fidelity of operation depends solely on the quality of the associated vacuum-tube equipment.

An automatic recording instrument embodying this principle is described in detail. It employs a beat-frequency oscillator as a source of variable frequency. A large number of experiments have verified the predicted superiority of the method. A number of representative records are presented.

INTRODUCTION

Almost all analyzers depend on some modification of the principle of resonance or selective response. Since Helmholz, it has been customary to employ a series of fixed resonators of known frequencies for the analysis of acoustic waves, the ear being depended on as a detector. Of a more recent date are analyzers employing electrical resonance. A condenser C arranged in series with an inductance L shows resonance at a frequency $n=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$. By changing the value of L or C, or both, the desired range of frequencies may be investigated.

This method has, however, a number of obvious difficulties. If the self-inductance be kept constant and it is desired to cover the ordinary voice range between, say, 20 and 10,000 cycles per second, a change in capacity in a ratio $\left(\frac{10{,}000}{20}\right)^2$ or 250,000 is required, which is highly impracticable. It then becomes necessary to vary the inductances also, and thus to employ a battery of inductances and condensers in connection with a complicated switching system (Reference 1.) The condensers in particular are subject to large temperature variations. This fact makes it impossible, or at least very difficult, to prevent overlapping or gaps between the various steps. The operation of the necessary switching equipment is an undesirable feature, and, furthermore, the response characteristic of such a simple electric resonator is inherently poor. To obtain any satisfaction in this respect a multiple system of tuning must be resorted to. This direct method has never proved to be of much practical value.

In order to avoid such difficulties, attempts have been made to transfer the sound spectrum to a higher frequency level—that is, to use a so-called modulated spectrum.

If a current is amplified through a nonlinear amplifier, the output contains a number of new components. The frequency and intensity of these components bear a certain relationship to the original constants. This new distorted spectrum offers some possibilities for the measurement of frequencies indirectly, that is, at higher or lower levels. The new spectrum, however, is crowded with a confusion of undesired frequencies.

The Bell Telephone Laboratories have recently published two reports relating to analyzers of this type. In one of these analyzers, described in a paper by Moore and Curtis (reference 2), the auxiliary current is changed in frequency throughout a range extending from 11,000 to 16,000 cycles, while the receiving unit is tuned to 11,000 cycles. In a second analyzer by A. G. Landeen (reference 3), the auxiliary current has a frequency around 40,000 and the receiver element is tuned to about 800 cycles. This instrument is adapted only to a frequency range in the neighborhood of 40,000 cycles per second. Since the Bell analyzers, employing the method of heterodyning the current to be analyzed with the current from a variable frequency source, are properly representative of such instruments, we shall devote the following section to a discussion of this method. We shall then present the theory of the hot-wire method of frequency analysis, give a description of the instrument, and also discuss the operating characteristics of the new recorder.

THEORETICAL CONSIDERATIONS REGARDING PREVIOUS METHODS FOR THE ANALYSIS OF COMPLEX CURRENTS

An approximate idea of the energy distribution in the modulated spectrum may be obtained from the following analysis.

Let the distortion be given by the relation

$$I_1 = A_0 + A_1 (V+X) + A_2 (V+X)^2 + A_3 (V+X)^3 + \ldots \quad (1)$$

where I_1 is the plate current of a vacuum tube, X and V are voltages impressed on the grid, and A_0, A_1, etc., are constants of the circuit. Let X represent some unknown voltage of any wave form and V a known voltage. If the unknown voltage X is impressed on another tube with its representative vector turned 180°, the resulting plate current in this tube circuit equals

$$I_2 = A_{0m} + A_{1m} (V-X) + A_{2m} (V-X)^2 + \ldots \quad (1')$$

With the two tubes working in a "push-pull" arrangement, the output is proportional to the difference of these two currents.

This difference equals

$$I_1 - I_2 = (A_0 - A_{0m}) + (A_1 - A_{1m})V + (A_1 + A_{1m})X + (A_2 - A_{2m})(V^2 + X^2) + 2(A_2 + A_{2m})VX + \ldots \quad (2)$$

This difference current, or, rather, some proportional quantity, is now usually fed into a resonant circuit for detection.

With careful matching of the circuit elements ($A_0 = A_{0m}$, $A_1 = A_{1m}$ etc.) the expression (2) reduces to

$$I_1 - I_2 = 2A_1X + 4A_2 VX + \ldots \quad (2')$$

This distorted spectrum contains the original frequencies as indicated by the term $2A_1X$.

The next term $4A_2VX$ exhibits a spectrum which is concentrated around the wave length corresponding to the known current V.

Let the frequency of the known current be n and let the frequencies of the complex current X be n_1, n_2, n_3, etc. The spectrum of the term $4A_2VX$ may then be shown to contain the frequencies

$$n+n_1,\ n+n_2,\ n+n_3, \text{ etc., and also}$$
$$n-n_1,\ n-n_2,\ n-n_3, \text{ etc.}$$

Let us assume that the original current represents a sound with a frequency range extending from 0 to 10,000 cycles per second. If the frequency n be kept constant, say, equal to 40,000 cycles per second, a displaced spectrum is obtained extending from 40,000 to 50,000, while a reversed issue extends from 40,000 down to 30,000. The situation is shown schematically in Figure 1.

Theoretically, we might analyze the displaced spectra B or C instead of the original spectrum A in Figure 1. Difficulties exist, however, with respect to the sharpness of the tuning. The internal resistance of the resonator circuit can not be made sufficiently small and the frequency discrimination is therefore poor. It is difficult to detect a small component at say, $n = 39{,}000$, if a large component is present at $n = 40{,}000$.

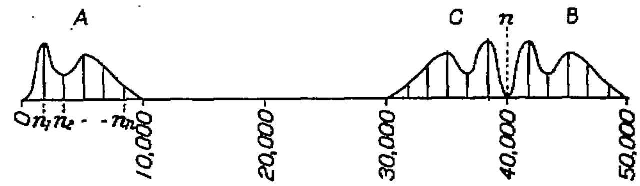

FIGURE 1.—Original and displaced spectra

An improved method has been devised by Moore and Curtis (reference 2), in which the known frequency V is varied while the receiving element remains tuned to a single fixed frequency. This scheme permits the employment of multiple-tuned circuits for better response. Moore and Curtis have employed a mechanical resonating element tuned to a frequency of 11,000. The original paper should be consulted to appreciate the difficulties inherent in this method.

The main objection to the method of employing a modulated spectrum for frequency analysis is the fact that the entire spectrum is crowded by unwanted frequencies. Equation (2′) represents the ideal condition only if terms of higher order than the second are negligible in equations (1) and (1′). A very limited range of the tube characteristic will satisfy this requirement. It is, however, also necessary that $A_0 = A_{0m}$, etc., which fact is obvious from (2). Anyone who has attempted to accomplish this matching will know that the design of an instrument of practical value on this basis is out of question.

PRINCIPLES OF THE HOT-WIRE FREQUENCY INDICATOR

We shall describe in the following pages a new type of instrument based on an entirely different phenomenon and shall also attempt to give its complete theory.

A remarkable property of the sine and cosine functions is that of orthogonality. If A and B are two such functions, the integral of their product equals zero if the integral is extended over a certain range, provided that A and B are of different order. If A and B are of the same order the integral is no longer zero, but has a definite value.

Applied to electrical phenomena, it is known that the resistance loss of an electric current is very dependent on the wave form. In particular it is recognized that several alternating currents may be transmitted over the same distribution system independently of each other as far as the resistance loss is concerned, provided their frequencies are all different. Let us assume that a number of frequencies are present and that one more current be added. If the new component happens to coincide vectorially with one of the original currents, an unexpectedly great resistance loss will result. It occurred to the author that this phenomenon could be employed as a basis for a new instrumental method of frequency analysis.

We will mention at once the peculiar fact that the very slowness of the response of a resistance wire, which usually is an undesirable feature, in the present case is even of value in certain respects. Only frequencies close to zero will appear in the temperature response, a property which may be compared to that of a low-pass filter, and the greater the thermal capacity the greater is the resolving power of the system.

Let two currents I and i be supplied simultaneously from two independent sources to a common resistance R. Let the current I be of any form whatsoever and let it be written in the form $\sum_n I_n$ where I_n refers to the instantaneous values of the harmonic components. Let i be a sine current of known frequency. The heat developed at any time in the resistance depends on the square of the instantaneous value of the current passing through it,

$$\frac{dH}{dt}=(I_1+I_2+I_3+\ \ldots\ +i)^2\,R.$$

Integrated over a certain time t, we obtain

$$H=R\int_0^t(I_1^2+I_2^2+I_3^2+2\,I_1\,I_2+2\,I_1\,I_3+2\,I_2\,I_3+i^2+2i\,I_1+2i\,I_2+2i\,I_3+\ \ldots)dt,$$

where H is expressed in calories or watt hours.

This integral may be simplified. We will not go into any mathematical detail, but just refer to current textbooks on trigonometric functions. The products I_1I_2, I_1I_3, etc., in the integrand may all be omitted, since they do not contribute anything to the value of the integral. Furthermore, if i happens to differ in frequency from any of the component parts of I, the products iI_1, iI_2, etc., are also of no effect in the integrand.

The amount of heat developed in the resistance is then simply

$$H=R\int_0^t(I_1^2+I_2^2+I_3^2+\ \ldots\ i^2)dt.$$

If, however, i is *identical in frequency and phase with one of the component* parts of the unknown current, say the n^{th} component I_n, the quantity of heat developed is *increased* by

$$2\,R\int_0^t i\,I_n\,dt.$$

If the currents i and I_n happen to be opposite in phase, the heat developed is *decreased* by

$$2\,R\int_0^t i\,I_n\,dt.$$

By adopting the symbols i and I_n as representing the *effective* values of the currents and assuming the currents to be in phase at $t=0$, we may write

$$\Delta H=4\,RiI_n\int_{-t}^{t}\sin\omega_1 t\sin\omega_2 t\,dt$$

or also

$$\Delta H=2\,RiI_n\int_{-t}^{t}[\cos\,(\omega_1-\omega_2)\,t-\cos\,(\omega_1+\omega_2)\,t]\,dt.$$

If this integral is extended over a sufficiently long time the average value of the integrand approaches zero. We are primarily interested in the particular case in which the two frequencies are nearly alike. The resistance wire will then exhibit a rather slow periodic temperature variation. The maximum quantity of "excess" heat developed at one time is obtained by extending the last integral from the time when $(\omega_1-\omega_2)\,t=-\frac{\pi}{2}$ to the time when $(\omega_1-\omega_2)\,t=+\frac{\pi}{2}$; that is, over the positive half period of the first term $\cos\,(\omega_1-\omega_2)\,t$ of the integrand. The second term $\cos\,(\omega_1+\omega_2)\,t$ contributes nothing and may be omitted.

The maximum value of ΔH is then equal to

$$2RiI_n\int_{pt=-\frac{\pi}{2}}^{+\frac{\pi}{2}}\cos\,pt\,dt=$$

$$2RiI_n\left[\frac{\sin\,pt}{p}\right]_{-\frac{\pi}{2}}^{+\frac{\pi}{2}}=2iI_nR\frac{2}{p}$$

where p is equal to $\omega_1-\omega_2$.

By introducing $T=\frac{\pi}{p}$ where T equals the half period, we obtain further

$$\Delta H=2iI_nR\frac{2}{\pi}T.$$

This value is equal to $\frac{2}{\pi}$ times the "excess" heat developed during the same time if both currents were exactly in phase.

Figure 2 indicates the effect. In (2A) both currents are in phase; in (2B) they are exactly out of phase. The former condition causes a permanent increase in the temperature of the resistance, the latter a decrease. In (2C) there exists a difference between the two imposed frequencies. The result is that the temperature of the resistance increases when the two currents are in phase and decreases when they are out of phase. In other words, the temperature shows a periodic change with a frequency equal to $\frac{p}{2\pi}$ vibrations per second.

This fact is of practical value. A resistance responds easily to "slow" changes, while the higher frequencies do not have any measurable effect. The actual response characteristic will be worked out in the following.

It is convenient to employ vacuum resistances of small thermal capacity. The filament of a vacuum tube is particularly suitable for our purpose. The rate

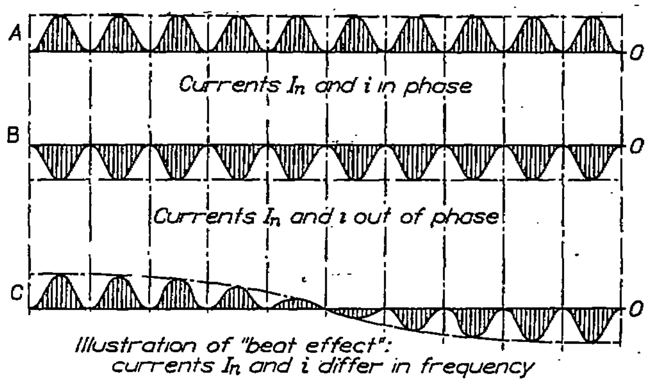

FIGURE 2.—Heating effects of the product $I_n i$

of energy supplied at temperature equilibrium is equal to $E_e = \alpha A T^4$, where α is the radiation constant, A the surface area of the resistance, and T the absolute temperature.

The assumption that all the heat is dissipated as radiation is nearly correct because the heat loss which is due to convection is negligible; furthermore, the temperature of the surroundings is small compared with the filament temperature T. In this equation E_e may be considered to express the constant (average) electric input to the vacuum resistance.

We know from the preceding analysis that

$$E_e = (\sum_n I_n{}^2 + i^2)\mathrm{R}$$

where I_n and i are the effective values of the currents.

Any instantaneous increase in the electric input will have *two* effects—it will increase the heat energy stored in the wire and it will increase the radiation loss.

The rate of "excess" heat developed may be expressed by a vector. The magnitude of this vector equals $\epsilon = 2RiI_n$ and its angular velocity is $\omega_1 - \omega_2 = p$.

Assume further that the wire shows an "excess" temperature of ΔT.

The excess radiation is consequently

$$\alpha A(T+\Delta T)^4 - \alpha A T^4 \text{ or}$$

$$\alpha A T^4 \left[4\frac{\Delta T}{T} + 6\left(\frac{\Delta T}{T}\right)^2 + 4\left(\frac{\Delta T}{T}\right)^3 + \left(\frac{\Delta T}{T}\right)^4\right].$$

It is seen that ΔT may be as much as 10 per cent of the value of T without causing appreciable inaccuracy by neglecting all but the first order term of $\frac{\Delta T}{T}$. The excess radiation may then be expressed as

$$4\alpha A T^3 \, \Delta T.$$

The rate of heat absorbed by the wire is simply $Vc\gamma \frac{\mathrm{d}}{\mathrm{d}t} \Delta T$ where c is the specific heat of the resistance, γ the density, and V the volume.

We may then write for the instantaneous values

$$\epsilon = 4\alpha A T^3 \Delta T + V\gamma c \frac{\mathrm{d}}{\mathrm{d}t} \Delta T \text{ or with } \Delta T = \Delta T_m \sin p\,t,$$

$$\epsilon = 4\alpha A T^3 \Delta T_m \sin p\,t + V\gamma c \Delta T_m p \cos p\,t.$$

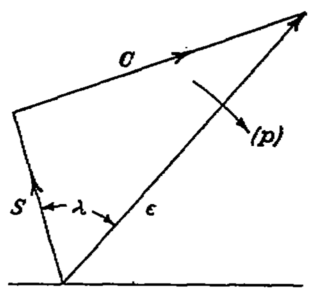

FIGURE 3.—Vector diagram representing the rate of "excess" energy input ϵ as the sum of the rate of "excess" radiation S and the rate of "excess" heat-storing C

The vector diagram in Figure 3 indicates the situation. The vector ϵ representing the "excess" energy input is ahead of the vector S representing the radiation. The angle of lag of the temperature is given by $\tan \lambda = \frac{C}{S} = \frac{V\gamma c p}{4\alpha A T^3}$, where C represents the "excess" heat absorbed by the heat-capacity of the wire.

We obtain from the diagram:

$$C^2 + S^2 = \epsilon^2$$

or
$$(V\gamma c \Delta T_m p)^2 + (4\alpha A T^3 \Delta T_m)^2 = (2RiI_n)^2$$

which gives

$$\Delta T_m = \frac{2RiI_n}{\sqrt{(V\gamma c p)^2 + (4\alpha A T^3)^2}}$$

$$= \frac{2RiI_n}{4\alpha A T^3 \sqrt{1 + \left(\frac{V\gamma c p}{4\alpha A T^3}\right)^2}}.$$

This expression can be brought into a more convenient form by introducing the relation

$$\alpha AT^4 = (\sum_n I_n^2 + i^2) R$$

$$\Delta T_m = \frac{2iI_n}{\sum_n I_n^2 + i^2} \frac{T}{4} \frac{1}{\sqrt{1 + \left(\frac{V\gamma cp}{4\alpha AT^3}\right)^2}}$$

or

$$\frac{\Delta T_m}{T} = \frac{1}{4} \frac{2iI_n}{\sum_n I_n^2 + i^2} \frac{1}{\sqrt{1+\tan^2 \lambda}} \tag{3}$$

The equation expresses the fact that the percentage increase in the temperature is equal to one-quarter times the percentage increase in the energy multiplied by a certain factor $(1+\tan^2\lambda)^{-1/2}$, or $\cos\lambda$.

It is beyond our powers to influence the constants of this relation with the exception of the last-mentioned factor involving the phase angle λ.

The phase angle is defined by

$$\tan\lambda = \frac{V\gamma c}{4\alpha AT^3} p.$$

Introducing the hydraulic radius $r = \frac{V}{A}$, the expression becomes

$$\tan\lambda = \frac{\gamma cr}{4\alpha T^3} p = \frac{1}{4} \kappa p,$$

where $\kappa = \frac{\gamma cr}{\alpha T^3}$.

The value of $\tan\lambda$ is directly proportional to the heat capacity per unit of volume γc, to the hydraulic radius r, and to the frequency p. It is inversely proportional to the radiation constant α, and to the third power of the absolute temperature T.

At this point attention is called to the peculiar condition, mentioned earlier in this report, that the heat capacity of the wire, usually an undesirable property, is responsible for the great selectivity attainable by the hot-wire method. It is seen that $\tan\lambda$ is zero when $p=0$ and that full "amplification" always is obtained in this case. The "width" of the response depends on κ. It may be of interest to mention the fact that a vacuum resistance a tenth of a millimeter in diameter is entirely too selective for all practical purposes.

There is not much choice as to the values of γ, c, and α. It was found that to decrease the constant κ in order to broaden the response sufficiently, it is desirable to employ a high temperature and to use a resistance of a hydraulic radius not more than about one-thousandth of a centimeter.

It may be noted that the constant κ in the above expression for $\tan\lambda$ corresponds to the time needed in heating the wire from absolute zero to the temperature T, assuming that all the energy liberated by the equilibrium value of the current is absorbed by the thermal capacity alone.

The initial rate of temperature rise at room temperature $\left(\frac{dT}{dt}\right)_r$ will actually furnish a quantitative indication of the constant κ.

The energy liberated by the equilibrium value of the current is equal to αT^4 $(A=1)$.

For the time interval dt we may then write

$$\alpha T^4 = \gamma cr \left(\frac{dT}{dt}\right)_r$$

or

$$\left(\frac{dT}{dt}\right)_r = \frac{\alpha T^4}{\gamma cr} = \frac{T}{\kappa}$$

where T is the final temperature.

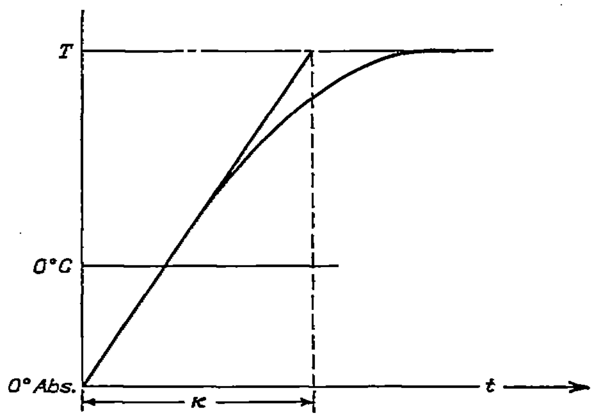

FIGURE 4.—Filament temperature against time

In Figure 4 the temperature has been plotted against time. The time needed to bring the temperature of the resistance up to near its equilibrium value will not differ greatly from the magnitude κ indicated in the figure. It is possible to obtain a fair estimate of this time by means of a stop watch. For the 301–A tube it is found to be about one-quarter of a second. A direct estimate by means of the formula $\kappa = \frac{\gamma cr}{\alpha T^3}$ yields approximately the same value.

It was found convenient to employ vacuum tubes to produce the effect. Not only is the filament of about the proper dimensions, but what is more important is the fact that the temperature fluctuations of the filament will produce corresponding changes in the flow of the plate current. The currents $\sum_n I_n$ and i are supplied to the filament, and beats occurring between the variable frequency current i and any of the unknown components I_n will manifest themselves as slow fluctuations in the plate current.

A low impedance milliammeter was selected as the most suitable indicator of the plate current. It is then advantageous to employ a step-down transformer as a coupling unit between tube and indicator.

Let us consider how this transformer affects the amplification. The current in the secondary of the transformer when working into a pure resistance equals

$$\Delta I_s = \frac{\Delta V}{nR_s} \frac{1}{\sqrt{1+\left(\frac{R}{pL}\right)^2}} \tag{4}$$

where ΔV is the equivalent voltage variation of the tube caused by the temperature variation ΔT_m of its filament, R is the total resistance of the primary circuit including the plate resistance of the tube, R_s is the secondary circuit resistance, n equals the turns ratio ($n>1$), and L equals the primary self-inductance of the transformer.

The above relation is correct only for $R_s=\infty$. The dependency of ΔI_s on p is, however, expressed with sufficient accuracy near $p=0$, and the simple form is retained mainly for the sake of simplicity in the following discussion.

For small fluctuations of the filament temperature it is also sufficiently accurate for the present purpose to assume *proportional* changes in the plate voltage. The factor of proportionality depends, of course, on the temperature level or operating range as shown in

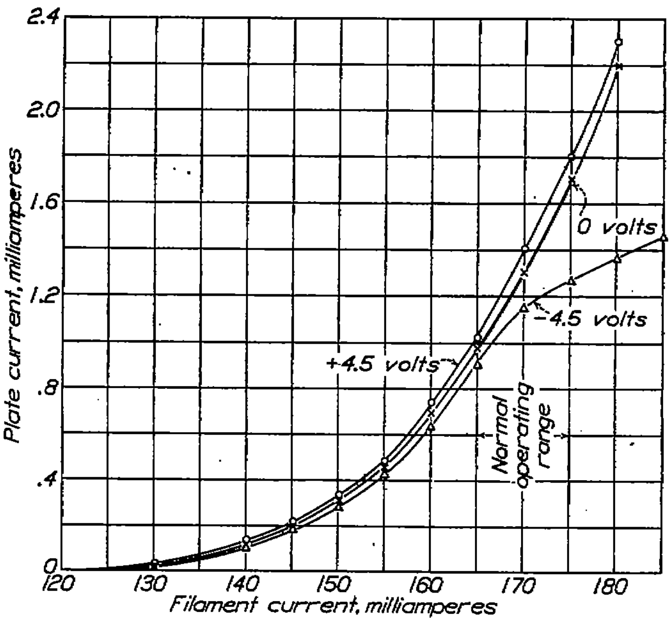

FIGURE 5.—Plate current of multiplier tubes as a function of filament current for different values of grid bias

Figure 5. Let us write $\Delta V=m\,\Delta T_m$ in equation (4), where m is a tube constant, and we get, by means of equations (3) and (4), the final result:

$$\Delta I_s=\frac{2mT}{nR_s4(\sum_n I_n^2+i^2)}\frac{1}{\sqrt{1+\left(\frac{R}{pL}\right)^2}}\frac{1}{\sqrt{1+\left(\frac{1}{4}\kappa p\right)^2}}iI_n$$

or

$$\Delta I_s=C_0\frac{1}{\sqrt{1+\left(\frac{R}{pL}\right)^2}}\frac{1}{\sqrt{1+\left(\frac{1}{4}\kappa p\right)^2}}iI_n \qquad (5)$$

where

$$C_0=\frac{2mT}{nR_s4(\sum_n I_n^2+i^2)}.$$

This equation expresses the fact that the output current ΔI_s is proportional to the product of the unknown component I_n and the known current i. The equation shows further that ΔI_s is a function of p, the difference of the frequencies of the two currents. The relation is of the form

$$\Delta I_s=C_0\,\Phi(p)\,i\,I_n.$$

The great convenience of an instrument based on this principle is immediately apparent to anyone familiar with the great complexity of difficulties encountered in the problem of sound analysis.

The most obvious advantage is the identical response at all frequencies. The equation shows that the response is dependent solely on the *difference* of the two frequencies and not on their absolute values. The "width" of the response pattern is, for instance, not greater at $n=8,000$ than it is at $n=80$.

The form of the function $\Phi(p)$ shows further that the response is rather critical. This is true whether a coupling transformer is employed or not. There is,

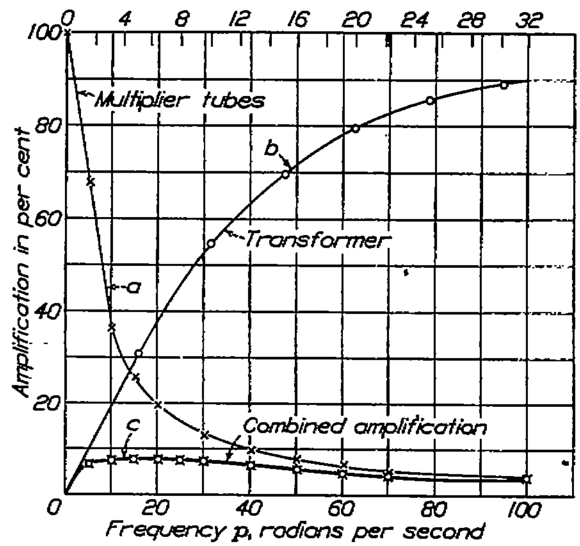

FIGURE 6.—Amplification of multiplier tubes and output transformer

however, a marked difference in the appearance of the two response patterns; in the latter case the function

$\Phi(p)$ is of the form $\dfrac{1}{\sqrt{1+\left(\frac{1}{4}\kappa p\right)^2}}$ (see fig. 6, curve a)

with the maximum response occurring at $p=0$. The sharpness of the response depends on κ, and in order to attain the greatest resolving power of the instrument, the resistance element should be made heavy. Theoretically, there is no limit to the amount of separation obtainable with such an arrangement.

With a coupling transformer the single response at $p=0$ is replaced by a pattern which has two peaks close to zero, one being an image of the other with respect to the zero point.

The response function $\Phi(p)$ is indicated by curve c in Figure 6 and by the records in Figures 16 and 17. In these figures the response intensity is plotted or recorded as ordinates against the frequency as abscissas. The maximum response occurs at a value of p equal to a few vibrations per second. By differentiation of $\Phi\ (p)$, this value is found to be

$$\Phi_m\ (p)=\frac{1}{1+\frac{\kappa R}{4L}}\ \text{occurring at the frequency}\ p=\sqrt{\frac{4R}{\kappa L}}.$$

With $\kappa=\frac{1}{4}$ second and the resistance-inductance ratio equal to about 20, the greatest response occurs at $p=18$ radians per second, or $f\cong 3$ vibrations per second. The maximum response $\Phi_m\ (p)$ equals about 45 per cent.

This apparent loss of response intensity is more than offset by the gain due to the more correct matching of the measuring device (increase in the value of C_s in equation (5)).

There is, however, another consideration to be taken into account in this connection. The low-frequency current $\triangle\ I$ is most properly indicated by an instrument working on the vibration principle. If the period is too close to zero, the time required to attain maximum indication becomes too great for practical purposes. It was found most satisfactory to employ an ordinary commercial milliammeter having a natural period of about three vibrations per second. The greatest response of the entire combination was thus expected at about this frequency. The distance between the two peaks of the response pattern, characteristic of this type of response, is actually close to six vibrations per second. (See figs. 16 and 17.)

DESCRIPTION OF INSTRUMENT FOR SOUND FREQUENCY ANALYSIS

A schematic diagram of the arrangement is shown in Figure 7. A commercial type beat-frequency oscillator is employed as a source of variable frequency current. It is noted that both the amplified sound current and the oscillator current are fed into a central unit termed "multiplier," the wiring diagram of which is shown in detail in Figure 8. This unit is the vital element of the instrument.

The sound is impressed across the primary of the transformer T_2. The secondary of this transformer is connected to the neutral point of the secondary winding of the beat-frequency-oscillator transformer T_3.

The combined output of the secondaries of these transformers is supplied to the *filaments* of the two audions M_1 and M_2. It is noted that the angle between the two voltages supplied to one of the audions differs by 180° from the angle between the voltages supplied to the other tube. In other words, if the phase angle difference for one tube is α then the phase angle difference for the other tubes is $\pi+\alpha$. The function of the high resistance potentiometers P_2 and P_3 is to permit a certain adjustment of the characteristics of the audions by means of their grid voltages. The total plate voltage variation of the audions M_1 and M_2 is supplied to the grid of a power tube V constituting a 1-stage amplifier. The output of this tube is matched to a low impedance milliammeter employed as a recorder by means of the step-down transformer T_4.

A close-up of the complete sound recorder is shown in the photograph, Figure 9.

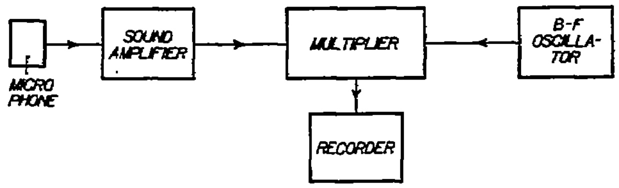

FIGURE 7.—Schematic diagram of sound-frequency analyzer

C is a condenser microphone.

A is the 4-stage microphone amplifier.

B is the beat-frequency oscillator with two stages additional amplification at A_B.

M is the multiplier, and

R a photographic recording device.

Figure 10 shows the recording device in more detail. Note that the film drum F and the dial D of the beat-frequency recorder are mechanically interconnected. The recorder contains, in addition to the cylindrical

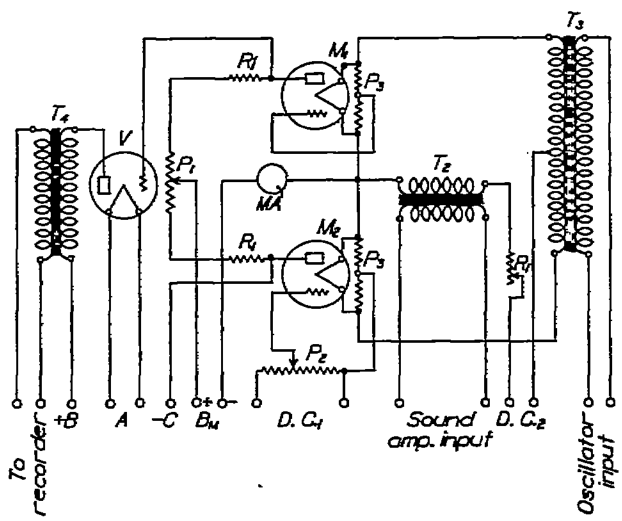

FIGURE 8.—Wiring diagram of the multiplier

film drum, a driving motor and the milliammeter which carries a small mirror. Light from a point source is reflected by this mirror onto the film drum. The film drum is thus travelling in step with the beat-frequency dial. A given point on the record will thus always correspond to the same frequency.

The original analyzer is shown in Figure 11. This instrument employed no output transformer after the multiplier; the recorder was connected in bridge arrangement across the tubes.

In the representative records reproduced in Figures 13 to 20 the ordinates indicate the intensity which is recorded against the frequency as abscissas. The time used for a complete record was 5 minutes. The operation is entirely automatic.

APPLICATIONS

Up to this time the frequency analyzer has been used largely for the recording of engine and propeller noises. These records will be published in a subsequent report. To show the variety of problems in which the analyzer is a convenient and very powerful means of investigation, a few representative records (figs. 13 to 20, inclusive) are included and will be discussed. The records are presented also for the purpose of indicating the properties of the analyzer.

The first record obtained with the original analyzer based on this new principle is shown in Figure 13. This record was obtained simply by letting the amplifier pick up the magnetic field from a 60-cycle toy transformer placed at some distance, with the field acting on the grid circuit of the first tube of the sound amplifier. It is noted that all harmonics up to about the twenty-fifth are recorded. This record and the diagram of the circuit in Figure 11 are shown only to illustrate the first step in the evolution of the present analyzer.

In contrast to this first record, Figure 14 shows a later record of the sound from a telephone receiver excited by a 60-cycle current. The sound output is distorted by an intentional overloading of the receiver. This record illustrates strikingly the great resolving power of the instrument. The record extends from 0 to 3,000 cycles per second. The harmonics appear over the entire range, and the frequency discrimination is just as good at 3,000 cycles per second as it is near 0. The slight crowding of the scale at the upper range is due to the fact that the beat-frequency oscillator employed did not have a straight-line frequency characteristic.

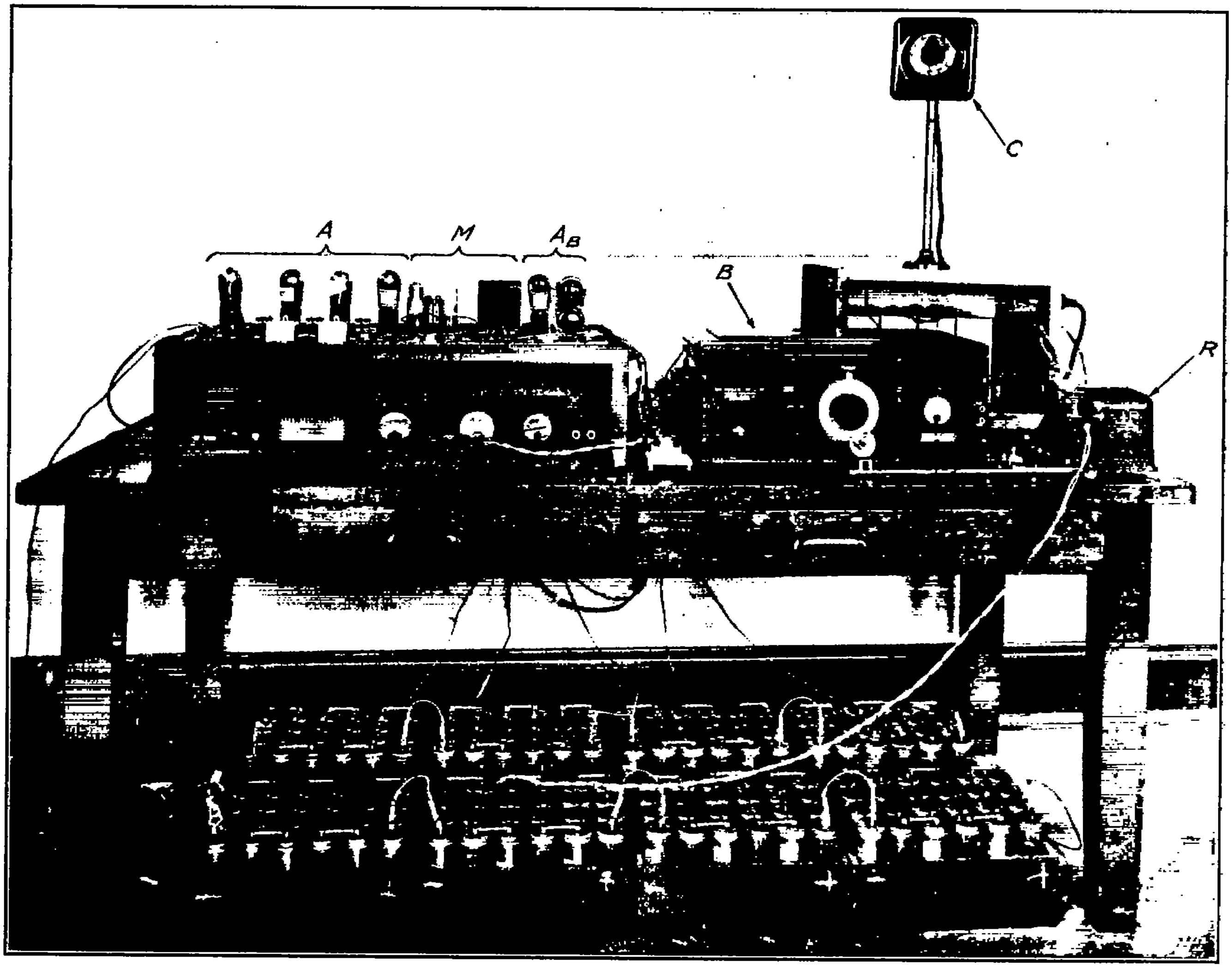

FIGURE 9.—The complete sound recorder in experimental form

Figure 15 shows the record obtained when the analyzer is connected to a 60-cycle source of voltage. Note the large 60-cycle component and the absence of harmonics.

Figures 16 and 17 show records similar to Figure 14 of the sound from an overloaded telephone receiver driven by a 60-cycle current. This record is taken over the lower frequency range to show the response pattern in more detail.

Figures 18 and 19 show the resolved sound issuing from a small 6-bladed propeller driven by a motor. The sound was picked up by a carbon button suspended close to the tip of the blades. The speed of the motor was in the first case 5,400 revolutions per minute and in the second case 3,000. Note that the fundamental frequencies appear at 540 and 300 cycles per second, respectively, corresponding to one impluse for each of the blades, or to a frequency six times greater than the number of revolutions per second. Several harmonics of this frequency are present. It is of particular interest to notice that no basic frequency corresponding to the number of revolutions of the propeller appears in these records.

By the simple expedient of placing the six blades at irregular intervals the basic frequency could be made to appear. The record shown in Figure 20 was obtained from a propeller with the blades spaced as in Figure 12. The basic frequency is present as expected. Rather surprising, however, is the fact that the fourth and the fifth harmonics are predominant, whereas the sixth is much smaller. By applying Fourier analysis to the problem it can be shown that the spacing was such as to favor the fourth and the fifth harmonics.

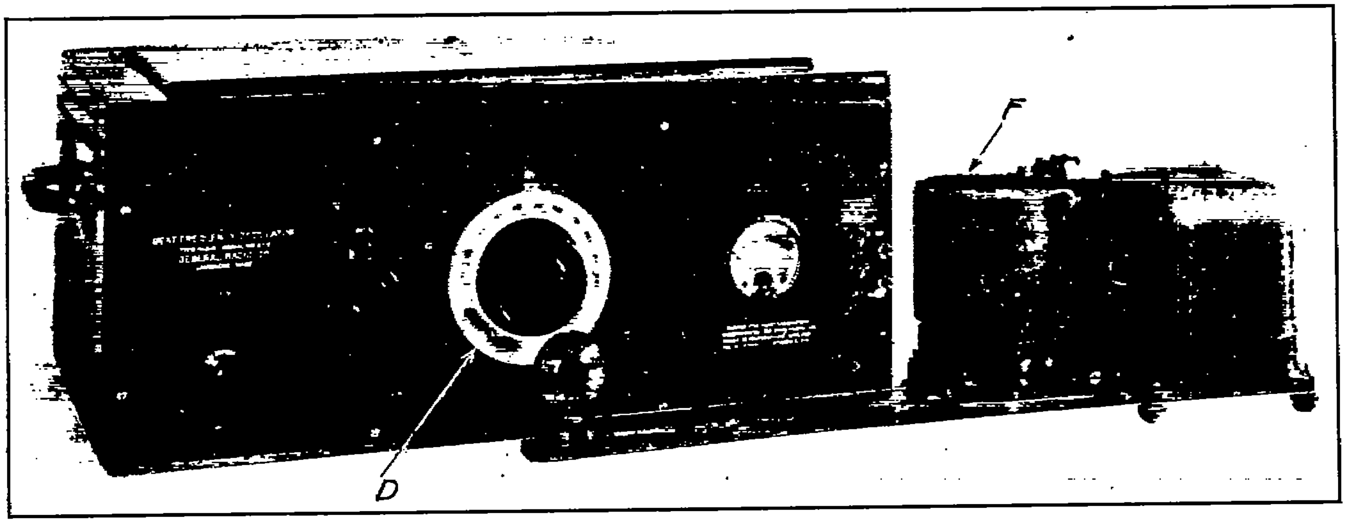

FIGURE 10.—The photographic recording device mechanically interconnected with the beat-frequency oscillator

DISCUSSION REGARDING THE OPERATING CHARACTERISTICS OF THE NEW ANALYZER

The current ΣI_n representing the unknown complex current supplies a certain energy $\Sigma I_n^2 R$ to the filament of the multiplier. The known current i from the beat-frequency source contributes the amount i^2R. The operating point on the characteristic curve obtained by plotting plate current against filament current (fig. 5) is thus dependent on the total energy level. The energy from the beat-frequency source drops off as zero frequency is approached, which would ordinarily result in a great decrease of sensitivity.

It is for this reason necessary, or at least desirable, to keep the filament temperature at approximately

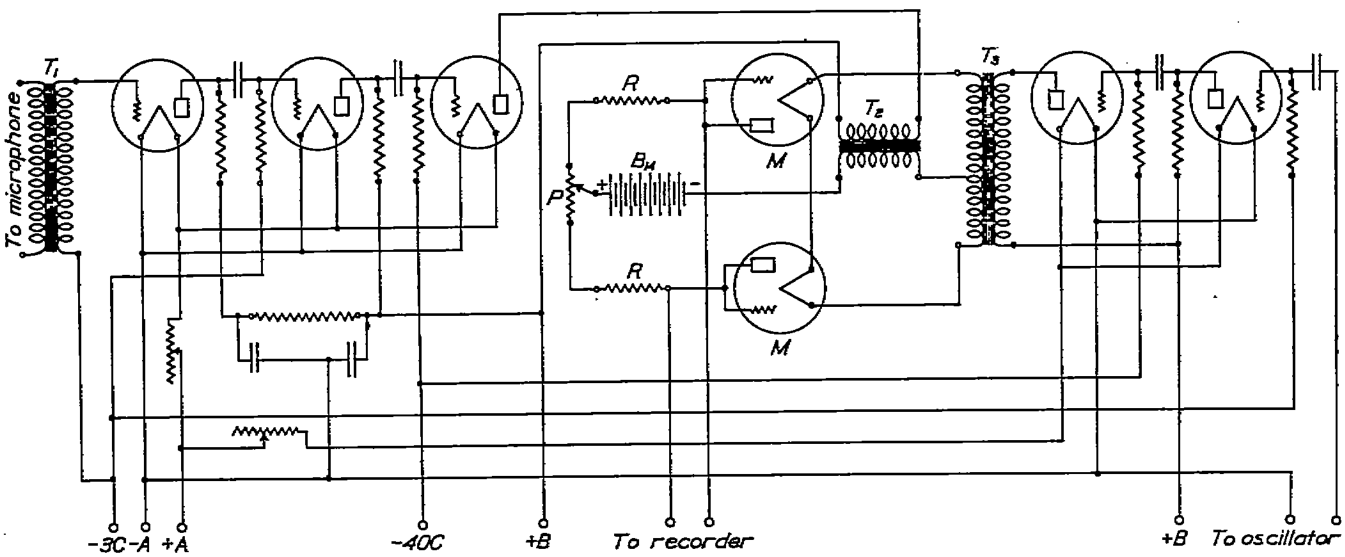

FIGURE 11.—Diagram of the original N. A. C. A. sound analyzer

the same level by means of a direct-current source as indicated by DC_2 in Figure 8. The ordinary operating range employed is shown in Figure 5.

Furthermore, the indication is directly proportional to the product $I_n i$. It is thus quite desirable to employ auxiliaries with straight-line characteristics. Most amplifiers possess this property between 50 and 5,000 cycles, even though the amplification falls off at very high and very low frequencies.

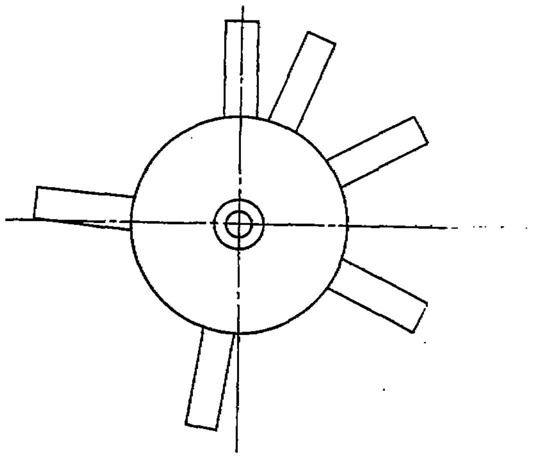

FIGURE 12.—Small 6-bladed propeller with irregular spacing

It is evident that the response intensity of the analyzer depends on the response of the auxiliaries and that this response can be made constant over the ordinary voice range.

The value of the new instrument depends, however, on the fact that *no harmonic distortion* takes place. The possibility of harmonic distortion is excluded by the very principles of this instrument. For a study of the serious difficulties encountered with the usual analyzers employing modulation the reader is referred to reference 2, pages 9 to 13. The present instrument is in a true sense a "multiplier." This term has been adopted because it properly reminds us of the fact that the response is proportional to the product $I_n i$ of the two currents. The fact that this product may be written

$$A \sin \alpha t B \sin \beta t = \frac{1}{2} A B \cos (\alpha - \beta) t - \frac{1}{2} A B \cos (\alpha + \beta) t,$$

together with the fact that the hot wire can respond only to the low frequency $(\alpha - \beta) t$, where $\alpha \cong \beta$, is in itself sufficient evidence that no harmonic distortion can be introduced by the hot wire. An experimental verification of the freedom from harmonic distortion is further given by the record (fig. 15) of a 60-cycle current.

In this connection, we are, of course, not referring to the distortion already present in the output of the microphone and the sound amplifier or in the variable frequency source. Modern equipment is, however, quite close to perfection in this respect.

LANGLEY MEMORIAL AERONAUTICAL LABORATORY,
NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS,
LANGLEY FIELD, VA., *March 23, 1931.*

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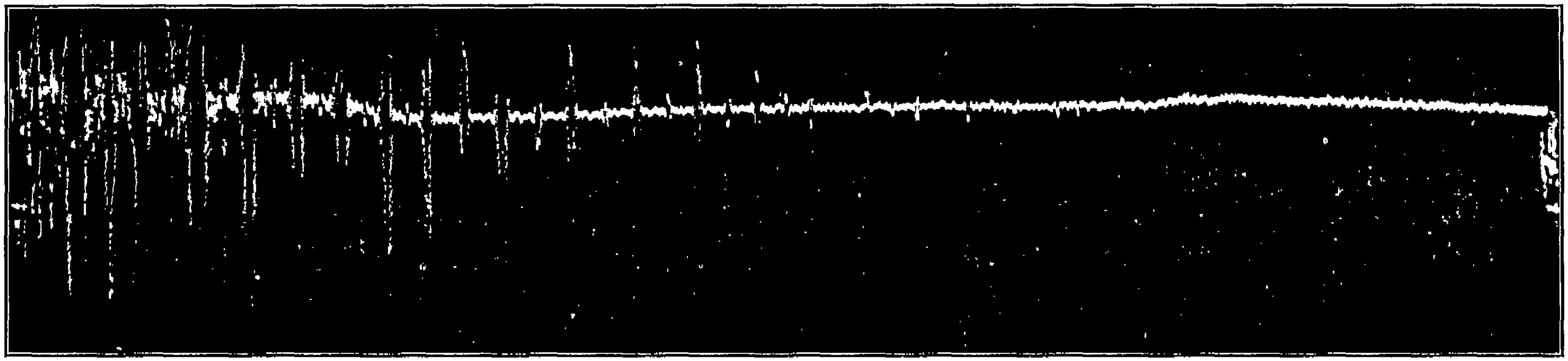


FIGURE 13.—First record obtained with original analyzer-record of field from a transformer

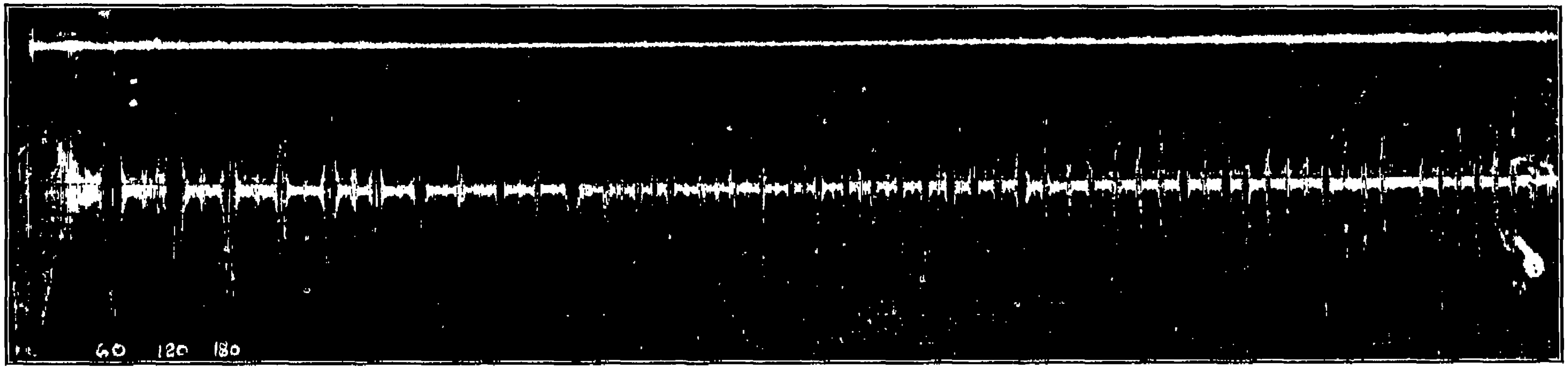


FIGURE 14.—Sound from a telephone receiver overexcited by a 60-cycle current

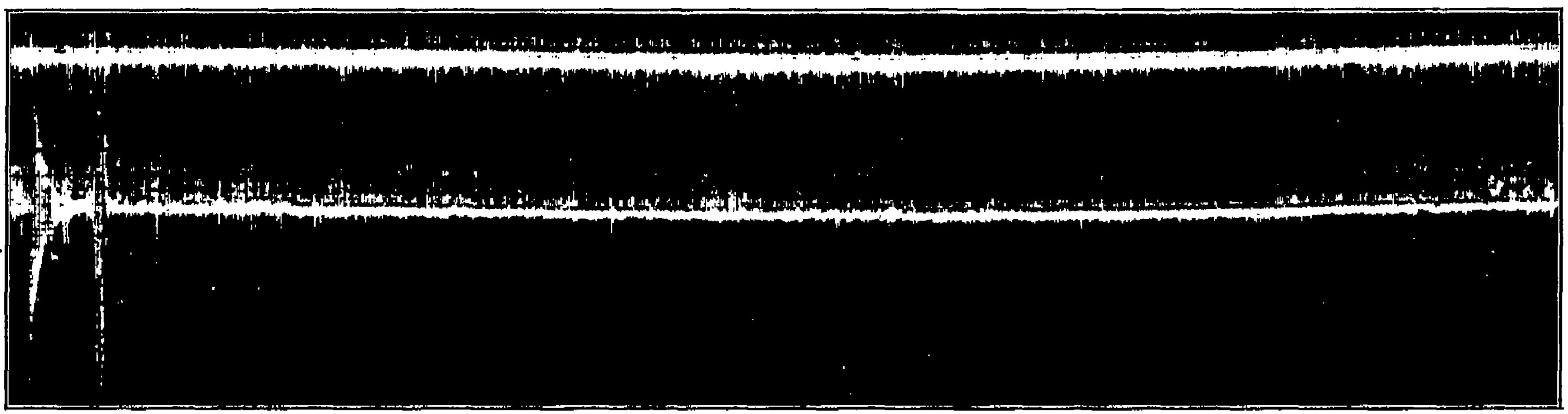
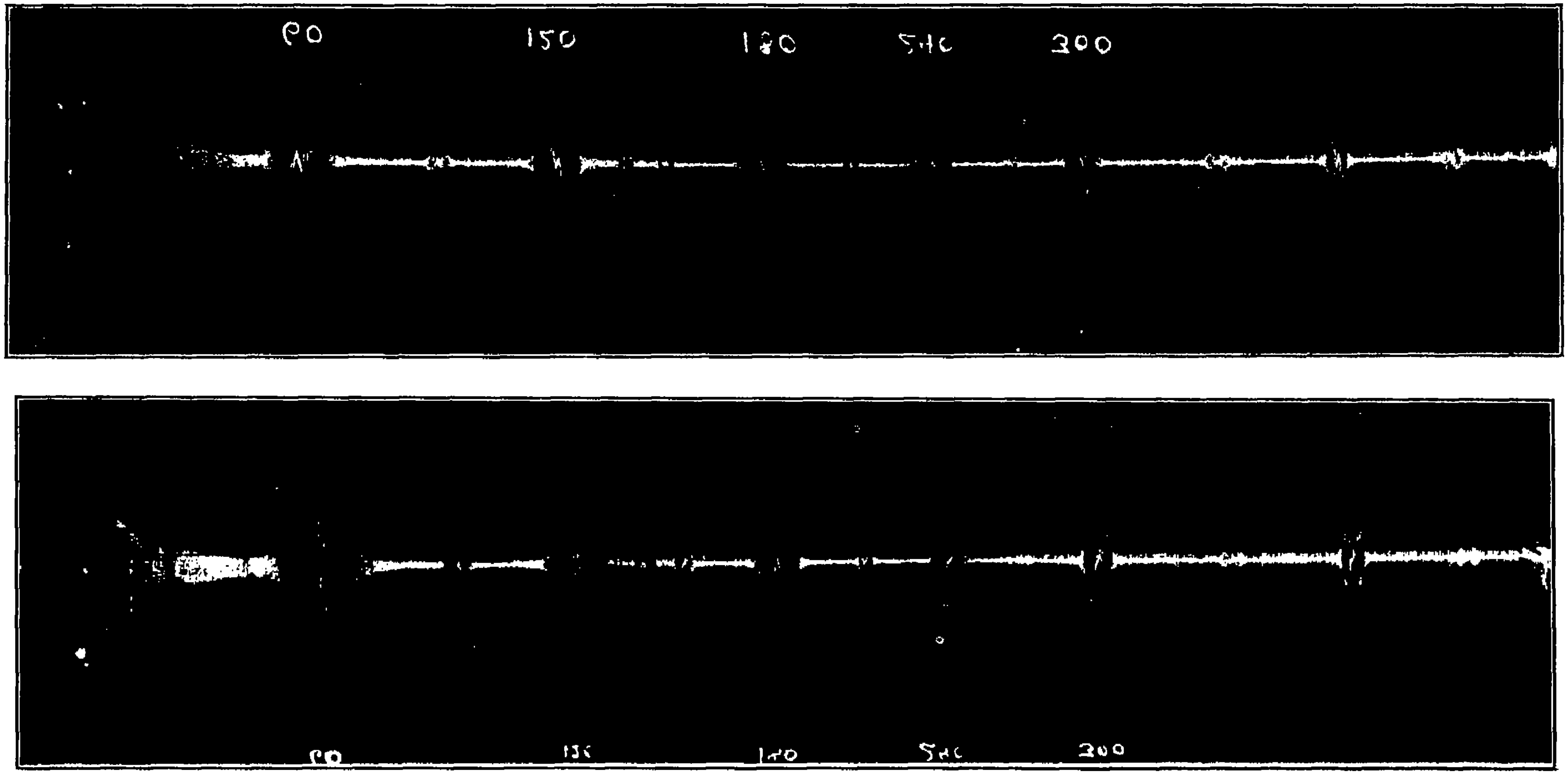
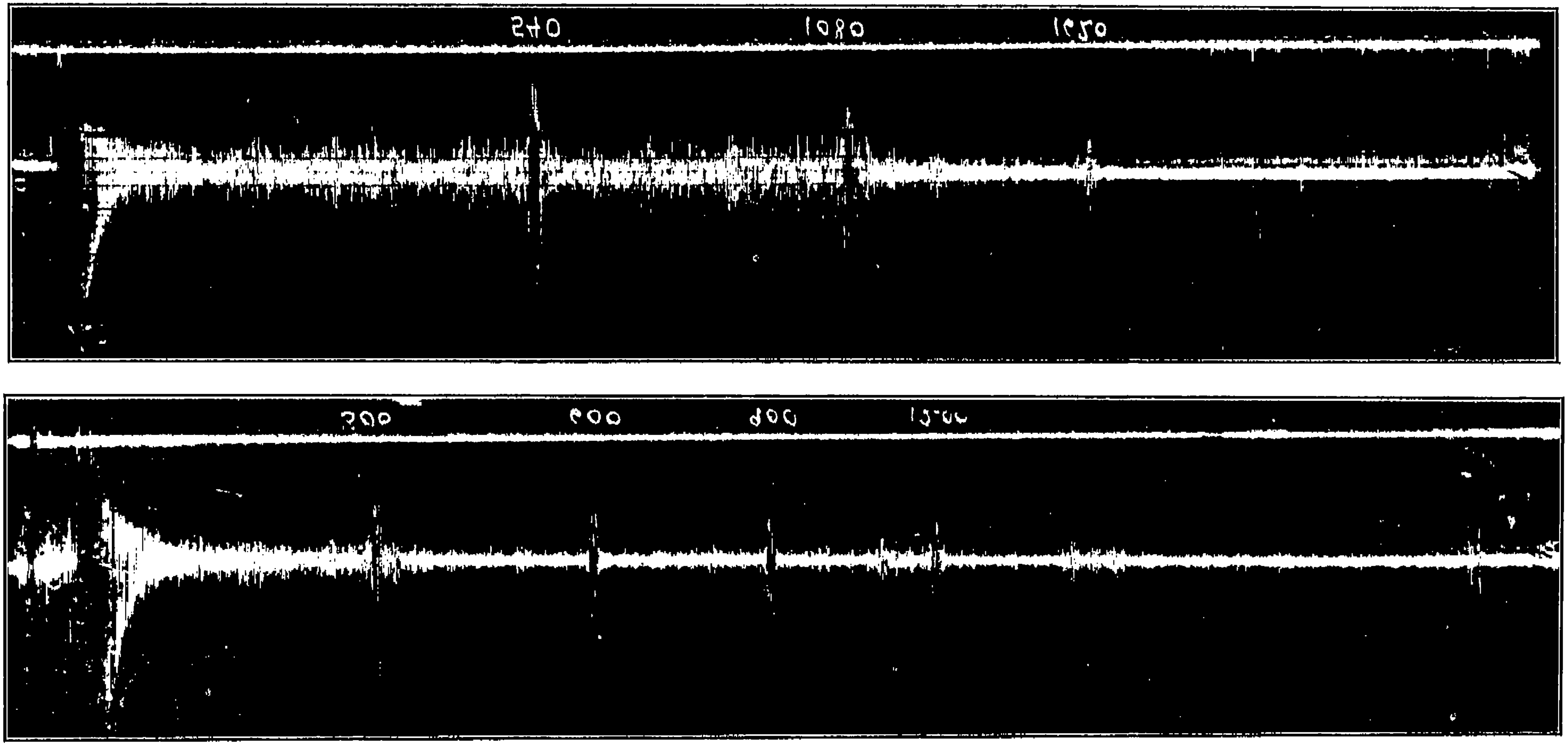


FIGURE 15.—Record of a 60-cycle source of voltage



FIGURES 16 and 17.—Sound from a telephone receiver overexcited by a 60-cycle current, taken over a lower frequency range to show response pattern in more detail



FIGURES 18 and 19.—Sound from small symmetrical 6-bladed propeller

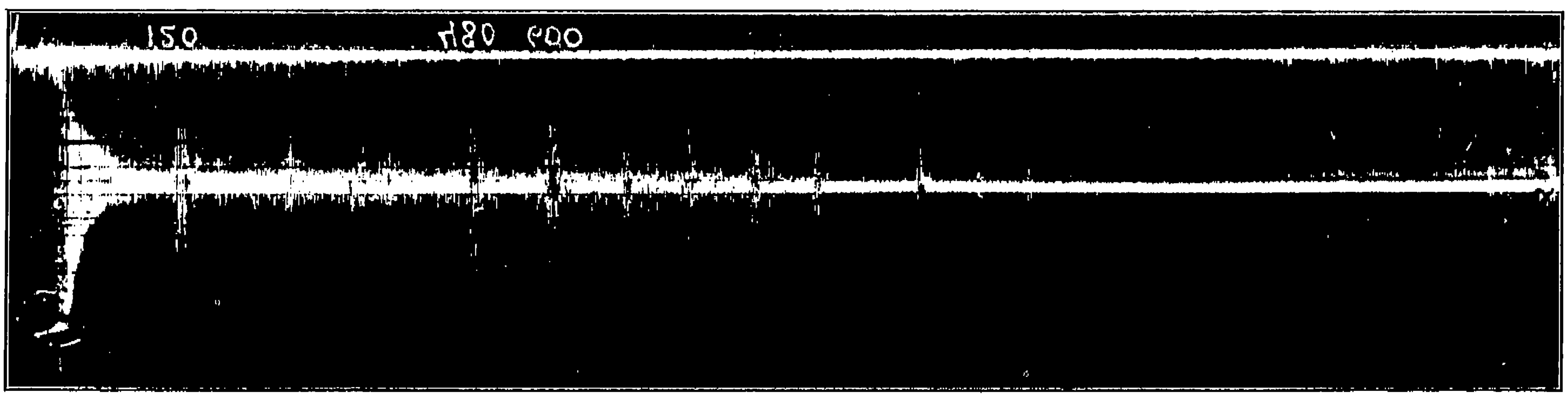


FIGURE 20.—Sound from small unsymmetrical 6-bladed propeller

PREFACE

During the last few years there has been a decided increase in the number of organizations working on the development of the compression-ignition engine for aircraft service. The advantages of the compression-ignition engine are threefold: It operates at a high-compression ratio, from 11 : 1 to 16 : 1, which results in a higher cycle efficiency and consequently a lower fuel consumption than the carburetor engine with a compression radio of 5 : 1 to 7 : 1; radio interference caused by an electrical ignition system is eliminated without shielding; the fire hazard is considerably lessened, because of the low volatility of the fuel. There are two disadvantages to the compression-ignition engine: Because of the high cylinder pressures the parts of the engine are more highly stressed than in the carburetor engine; the allowable time for the injection and its mixture with the air in the combustion chamber is extremely short, so that it is difficult to obtain a high combustion efficiency.

One of the important problems in the design of a high-speed compression-ignition engine suitable for aircraft service is the design of the fuel injection system. There has been considerable work done both here and abroad to determine the operating characteristics of several types of systems. Until the latter part of 1929, however, there was little materia available on the theory of fuel injection systems. Since then Doctor Sass has published material on the adaptation of the Allievi theory of water hammer to fuel injection pumps for compression-ignition engines. Before the publication of Doctor Sass's work, a series of tests had been made at the Langley Memorial Aeronautical Laboratory in which the instantaneous discharge pressures from a fuel injection pump were measured. With the data obtained in these tests and the theory presented by Doctor Sass, as well as additional analyses, it was thought advisable to write a report incorporating this material for use in the design of injection systems for high-speed compression-ignition engines.

In presenting this material in such a manner that it can be used in actual design, the author has attempted to give the mathematical analyses in such a way that they will be readily understood. Numerical examples are presented in considerable detail and emphasis is laid on the necessity of having all equations satisfied dimensionally. Although at times it may seem as though too much detail is included, the author felt that it was better to present some material that may seem obvious than to shorten the work so that the derivations could not be easily followed.